Revisiting non-Gaussianity of multiple-field inflation from the field equation

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Abstract

In the present paper, we study the non-Gaussianity of multiple-field inflation model using the method of the field equation. We start from reviewing the background and the perturbation theory of multiple-field inflation, and then derive the Klein-Gorden equation for the perturbations at second order. Afterward, we calculate the tree-level bispectrum of the fields' perturbations and finally give the corresponding parameter f_{NL} for the curvature perturbation ζ in virtue of the δN formalism. We also compare our result with the one already obtained from the Lagrangian formalism, and find they are consistent. This work may help us understand perturbation theory of inflation more deeply.

1 Introduction

It is suggested that our universe has undergone an inflationary stage in the early time. This scenario helps us understand why our universe are so flat and isotropic, and also provides a possible solution to the monopole problem in the hot Big-Bang cosmology [1, 2, 3]. The most efficient model of inflation is driven by a single scalar field which rolls down along its potential very slowly. This model generically predicts a scale-invariant powr spectrum and so is able to explain the formation of the large scale structure. This expectation has already been confirmed by the 5-year WMAP data [4] which is the latest observation of cosmic microwave background radiation (CMBR). Although the single field inflation model has obtained fruitful achievements, we still need to explore more on this theory.

A significant lesson is to investigate its higher order perturbations. There has been a number of literature studying the behavior of higher order perturbations in inflation models (see [5] for an excellent pioneer work, and see [6] for a good review on this issue). From the viewpoint of statistic dynamics, these higher order perturbations are usually related to n ($n > 2$) point correlators. So if these correlators indeed exist, there must be non-Gaussianity in the early universe. The non-Gaussianity is a very important issue worth studing. Since the non-Gaussianity has many features which can be observed by experiments, such as its magnitude, shape, running and so on, it encodes plentiful information about the early universe. We are able to learn what has happened since that time if we detect it. For example, we have already known that the primordial bispectrum of single scalar field inflation model is too small to be observed [5]. However, there are implications of non-Gaussianity which value may be large from astronomical data [4, 7] recently. If this is confirmed, the usual inflation models, especially chaotic inflation, will suffer a great challenge from the experiments. Another example is, that different models usually have different predictions about non-Gaussianity [6, 8, 9, 10, 11, 12, 13, 14, 15, 16], and thus the non-Gaussianity may help us to discriminate these models.

One may be interested in how to produce large non-Gaussianity in inflation. Firstly, let us think about why the non-Gaussianity of single scalar field inflation model is so small. Ordinarily, the non-Gaussian effect comes from the interactions

of the perturbation variables in the canonical case that we choose the Bunch-Davis vacuum. In the single scalar field inflation model, the interaction terms of the perturbation variables are strongly suppressed by slow roll parameters. However, if we modify the lagrangian of the inflaton to be non-canonical, such as DBI inflation [9, 14] and K-inflation [10], it is possible to obtain a large value of non-Gaussianity. Another way to obtain large bispectrum is to change the initial condition of Gaussian statistic, e.g. a thermal initial condition [15].

The above arguments are valid when we only consider the adiabatic perturbations. It is feasible since the perturbations generated in single scalar field inflation model are always highly adiabatic and the curvature perturbation ζ is conserved on large scale. However, this picture is changed when the inflation is driven by multiple fields [17, 18]. When we introduce multiple fields, they will generate a large amount of entropy fluctuations which are converted into the curvature perturbations at later time. This scenario can result in large non-Gaussianity of local form. For example, the curvaton mechanism [19, 20] and the in-homogenous reheating scenario [21, 22, 23] are able to produce large non-Gaussianity of local form. Therefore, it is meaningful to study the bispectrum of multiple-field inflation with both the magnitude and the shape in detail.

Interestingly, the method of calculating the primordial non-Gaussianity is not unique, and a number of methods have been proposed in literature. These methods possess different advantages in different occasions. The most direct formalism was developed by [24] in which the authors calculated the second order perturbations from the Einstein equations; another useful formalism was called Lagrangian formalism [5] which derived the interaction terms of curvature perturbations in the Lagrangian. Both the two approaches are able to calculate the magnitude and the shape of the non-Gaussianity of which the local one is the most interested in observations. Moreover, a so-called δN formalism [25] has been proposed to calculate the local non-Gaussianity specifically. This method greatly simplified the calculation of non-Gaussianity. Some pioneer works on the non-Gaussianity of multiple-field inflation have been done by using different methods. For example, the local form of non-Gaussianity in two-field inflation is shown by [26] based on δN formalism; the shape of non-Gaussianity in multiple-field inflation is given by the Lagrangian formalism in [27]. Recently, a

remarkable work has been done by [28] in which the authors have used second-order Klein-Gordon equation [29, 30] to calculate the non-Gaussianity, which is consistent with the Lagrangian formalism. The method of field equation can directly derive the non-Gaussianity from equation of motion, without assuming an effective action principle. We in this paper extend the field equation formalism to the multiple-field inflation and calculate the non-Gaussianity. In the derivation, we assume that there are $\mathcal{N}$ scalar fields $\phi^I,\ \phi^J,\ \cdots$ in the period of inflation, and the potential V of the scalar fields depends on them. We take the natural unit $M_P \equiv (8\pi G)^{-1/2} = 1$ in this paper.

Our paper is organized as follows. In Section §2, we review the background evolution of the multiple-field inflation, and define the slow-roll parameters. In section §3, the quantum theory of the first order perturbations is discussed. By means of the canonical method, we quantize the perturbations of scalar fields, and present the Green's functions. In Section §4, we derive the second order Klein-Gordon equation directly from the action, and so the second order perturbations of scalar fields are obtained by the Green's function. Section §5 presents the main result of our paper which shows that there are different source terms contributing to the three-point correlator of scalar fluctuations. In Section §6, we review the δN formalism, and calculate the nonlinear parameter $f_{\rm NL}$. Conclusions and discussions are summarized in the last section.

2 The background in multiple-field inflation

In this section, we show the field equations in the background, and define some slow roll parameters in multiple-field inflation. The background is assumed to be the Friedmann-Robertson-Walker (FRW) spacetime, and the action takes the form

$$\mathrm{d}s^2 \ = \ -\mathrm{d}t^2 + a(t)^2 \delta_{ij}\, \mathrm{d}x^i\, \mathrm{d}x^j \ , \tag{1}$$

where $a(t)$ is the scale factor. In some cases, it is convenient to use conformal time η, which is defined as $\eta \equiv \int_t^\infty \mathrm{d}t'/a(t')$. And to the leading order of slow roll approximation, $\eta \sim -\frac{1}{aH}$ in the period of inflation.

The equation of scalar field takes the form

$$\phi_0^{I\prime\prime} + 2\mathcal{H}\phi_0^{I\prime} + V_{,I} = 0 \ , \tag{2}$$

where I denotes different scalar fields, prime denotes $\frac{\mathrm{d}}{\mathrm{d}\,\eta}$, $V_{,I}$ is the shorthand for $\frac{\mathrm{d}V}{\mathrm{d}\phi^I}$, $\mathcal{H} \equiv a'/a$ is the conformal Hubble scale, and the metric of the field space is assumed to be δ_{IJ}.

The 0-0 component of Einstein equations gives the so-called Friedmann equation,

$$3\mathcal{H}^2 = \frac{1}{2}\delta_{IJ}\phi_0^{I\prime}\phi_0^{J\prime} + a^2 V(\phi_0) \ . \tag{3}$$

And from the i-j component of Einstein equations, we have

$$\mathcal{H}^2 + 2\mathcal{H}' = -\frac{1}{2}\delta_{IJ}\phi_0^{I\prime}\phi_0^{J\prime} + a^2 V(\phi_0) \ , \tag{4}$$

where the repeated up index and down index represent summation.

As in single field inflation, the potential should satisfy the slow roll condition due to the constraint from the observation. It requires that the velocity and acceleration of inflaton rolling down the potential are very small. In the multiple-field inflation, we use the slow roll matrix

$$\epsilon^{IJ} = \frac{\dot{\phi}_0^I \dot{\phi}_0^J}{2H^2} = \frac{\phi_0^{I\prime}\phi_0^{J\prime}}{2\mathcal{H}^2} = \epsilon^I \epsilon^J, \tag{5}$$

where

$$\epsilon^I = \frac{\dot{\phi}_0^I}{\sqrt{2}H} \ , \tag{6}$$

and the trace of the slow roll matrix $\mathrm{tr}\,\epsilon^{IJ}$ is the standard slow roll parameter $\epsilon = -\dot{H}/H^2$. In general situation, the order of these slow roll parameters are estimated as

$$\epsilon^{IJ} \sim \mathcal{O}(\frac{\epsilon}{\mathcal{N}}), \ \epsilon^I \sim \mathcal{O}(\sqrt{\frac{\epsilon}{\mathcal{N}}}) \ . \tag{7}$$

To generalize the single field inflation, we introduce the second slow roll matrix,

$$\eta^{IJ} = \frac{\ddot{\phi}^I \dot{\phi}^J + \dot{\phi}^J \ddot{\phi}^J}{4H\dot{H}} \ . \tag{8}$$

The diagonal element of this matrix is the slow parameter in the single field inflation $\eta^{\phi\phi} = -\frac{\ddot{\phi}}{H\dot{\phi}} = \eta$.

3 The First-order perturbation in the uniform curvature gauge

When we compute the perturbation of inflation, the quantity is usually changed with the coordinate transformation. In order to discuss the real physical freedoms in the inflationary perturbation theory, we should select a gauge [31]. Fixing a gauge means choosing a coordinate system. Different gauges are equivalent in physics. In this section we select the uniform curvature gauge and discuss the first order perturbation of real physical freedoms. It is convenient to study in ADM formalism and the metric can be expressed as

$$\mathrm{d}\, s^2 = -\mathrm{N}^2 \mathrm{d}t^2 + h_{ij}(\mathrm{d}x^i + \mathrm{N}^i dt)(\mathrm{d}x^j + \mathrm{N}^j \mathrm{d}t), \tag{9}$$

so the action is

$$\begin{aligned} S &= -\frac{1}{2}\int \mathrm{N}\sqrt{h}\,\left(\delta_{IJ}h^{ij}\partial_i\phi^I\partial_j\phi^J - 2V(\phi)\right) \\ &\quad +\frac{1}{2}\int \mathrm{N}^{-1}\sqrt{h}\,\left(E_{ij}E^{ij} - E^2 + \delta_{IJ}(\dot{\phi}^I - \mathrm{N}^j\partial_j\phi^I)(\dot{\phi}^J - \mathrm{N}^j\partial_j\phi^J)\right)\ , \end{aligned} \tag{10}$$

where $\mathrm{N}^{-1}E_{ij}$ is the extrinsic curvature, $E = E^i_i$. We select the uniform curvature gauge, in which the Ricci curvature is zero at the same coordinate t and $h_{ij} = a^2(t)\delta_{ij}$. The two scalar perturbations from the metric perturbation can be expressed by the lapse N, and shift N^i. The lapse N, and shift N^i are Lagrangian multipliers. Thus the physical freedoms can be expressed by the $\mathcal{N}$ scalar perturbations $\delta\phi^I$ in the uniform curvature gauge.

As in the single field, the scalar perturbation can be expanded in powers of the gaussian perturbation $\delta\phi^I_1$,

$$\delta\phi^I = \delta\phi^I_1 + \frac{1}{2}\delta\phi^I_2 + \cdots + \frac{1}{n!}\delta\phi^I_n + \cdots\,. \tag{11}$$

The closer the primordial scalar perturbation is to gaussian statistics, the better the expansion is.

Since $\delta\phi^I_1$ obeys the gaussian statistics, the equation of motion of $\delta\phi^I_1$ is linear. After some simplification of (10), the second order action takes the form

$$S_2 = \frac{1}{2}\int \mathrm{d}\eta\, \mathrm{d}^3x\; a^2\left(\delta_{IJ}\delta\phi^{I\prime}_1\delta\phi^{J\prime}_1 - \delta_{IJ}\partial\delta\phi^I_1\partial\delta\phi^J_1\right)\ , \tag{12}$$

where $\partial\delta\phi_1^I\partial\delta\phi_1^J$ is the shorthand for the scalar product $\delta^{ij}\partial_i\delta\phi_1^I\partial_j\delta\phi_1^J$. Then the field equation of scalar field $\delta\phi^I$ for the Fourier mode is

$$\delta\phi_1^{I\prime\prime} + 2\mathcal{H}\delta\phi_1^{I\prime} + k^2\delta\phi_1^I = 0, \tag{13}$$

The classical field is quantized by the canonical method,

$$\delta\hat{\phi}_1^I(\mathbf{x},\eta) = \int \frac{\mathrm{d}^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \{a_{\mathbf{k}}^{I\dagger}\theta_k^I(\eta) + a_{-\mathbf{k}}^I\bar{\theta}_k^I(\eta)\} \ , \tag{14}$$

where θ_k^I, $\bar{\theta}_k^I$ are massless scalar fields in momentum space. The normalization of the terms is determined by the commutative relation between scalar field and its canonical momentum, and the commutative relation between the creation and annihilation operator

$$[a_{\mathbf{k}}^I, a_{\mathbf{k}'}^{J\dagger}] = (2\pi)^3\delta^{IJ}\delta(\mathbf{k}-\mathbf{k}') \ . \tag{15}$$

In the Bunch-Davies vacuum, the normalized scalar field is [32],

$$\theta_k^I = \frac{H}{\sqrt{2k^3}}(1-ik\eta)e^{ik\eta} \ . \tag{16}$$

Since the value of θ_k^I is independent of I, we omit the index I in θ_k^I afterwards. The two-point correlator of scalar fields is

$$\begin{aligned}
\langle\delta\phi_1^I(\mathbf{k},\eta)\delta\phi_1^J(\mathbf{k}',\eta')\rangle &= (2\pi)^3\delta^{IJ}\delta(\mathbf{k}+\mathbf{k}')\bar{\theta}_k(\eta)\theta_k(\eta') \\
&\sim (2\pi)^3\delta^{IJ}\delta(\mathbf{k}+\mathbf{k}')\frac{H^2}{2k^3} \qquad for \quad k\eta \ll 1 \\
&= (2\pi)^3\delta^{IJ}\delta(\mathbf{k}+\mathbf{k}')\frac{2\pi^2}{k^3}\mathrm{P}(k) \ ,
\end{aligned} \tag{17}$$

where $\mathrm{P}(k) = \frac{H^2}{4\pi^2}$ is the so-called power spectrum of scalar field. And the retarded Green's function in momentum space takes the form,

$$Gr_k(\eta,\tau) = ia(\tau)^2 \times \begin{cases} 0 & \eta < \tau \\ \theta_k(\tau)\bar{\theta}_k(\eta) - \bar{\theta}_k(\tau)\theta_k(\eta) & \eta > \tau \end{cases} \ . \tag{18}$$

Using the Green's function, the second order field equation can be solved as a linear function with the source term.

4 The second-order Klein-Gordon equation

In this section, we derive the second-order Klein-Gordon equation from the multiple-field action (10). The situation of single field is given by [30]. Expanding the action (10), it includes the terms of all the scalar fields and scalar perturbations from the metric. The lapse and the shift in the action are determined since they are Lagrangian multipliers without dynamics effect. Finally they are eliminated from the action which only contains the second-order perturbation of scalar fields $\delta\phi_2^I$. The part of the action quadratic in $\delta\phi_2^I$ can be expressed in conformal time,

$$S_2 = \frac{1}{8}\int \mathrm{d}\eta\, \mathrm{d}^3x\; a^2 \left(\delta_{IJ}\delta\phi_2^{I\,\prime}\delta\phi_2^{J\,\prime} - \delta_{IJ}\partial\delta\phi_2^I\partial\delta\phi_2^J\right) \ . \tag{19}$$

and the cubic term in the slow roll approximation [27] is

$$\begin{aligned} S_3 \;\; = \;\; & \int \mathrm{d}\eta \mathrm{d}^3x a^2 [\frac{1}{3!}V_{,IJK}\delta\phi_2^I\delta\phi_1^J\delta\phi_1^K + \delta_{IJ}\delta_{MN}\frac{\delta\phi_0^{M\,\prime}}{4\mathcal{H}}\delta\phi_2^{I\,\prime}\partial\nabla^{-2}(\delta\phi_1^{N\,\prime})\partial\delta\phi_1^J - \\ & \delta_{IJ}\delta_{MN}\frac{\delta\phi_0^{M\,\prime}}{8\mathcal{H}}\delta\phi_2^N\delta\phi_1^{I\,\prime}\phi_1^{J\,\prime} - \delta_{IJ}\delta_{MN}\frac{\delta\phi_0^{M\,\prime}}{8\mathcal{H}}\delta\phi_2^N\partial\delta\phi_1^I\partial\delta\phi_1^J] + perms \ , \end{aligned} \tag{20}$$

where the permutations represent swapping the $\delta\phi_2$ in other possible positions. The term containing $V_{,IJK}$ in the action is not neglected by the slow-roll approximation, because it may contribute large effect in non-Gaussianity [23].

Variation $\delta S/\delta(\delta\phi_2^I) = 0$ gives the field equation. All the surface terms are neglected, which requires that $\delta(\delta\phi_1^I)$ vanishes in the boundary, and the equation of motion for $\delta\phi_1^I$ simplifies the result further. The final result is

$$\begin{aligned} & \delta\phi_2^{I\,\prime\prime} + 2\mathcal{H}\delta\phi_2^{I\,\prime} + k^2\delta\phi_2^I \\ = \;\; & (-a^2V_{,IJK}\delta\phi_1^J\delta\phi_1^K) + \frac{\delta\phi_0^{M\,\prime}}{\mathcal{H}}(-2\delta_{MN}\partial\nabla^{-2}\delta\phi_1^{N\,\prime}\partial\delta\phi_1^{I\,\prime} + 2\delta_{MN}\delta\phi_1^N\nabla^2\delta\phi_1^I) \\ & + \frac{\delta\phi_0^{I\,\prime}}{\mathcal{H}}[-\frac{1}{2}\delta_{MN}\delta\phi_1^{M\,\prime}\delta\phi_1^{N\,\prime} - \frac{1}{2}\delta_{MN}\partial\delta\phi_1^M\partial\delta\phi_1^N + \delta_{MN}\nabla^{-2}(\partial\nabla^2\delta\phi_1^M\partial\delta\phi_1^N \\ & + \nabla^2\delta\phi_1^M\nabla^2\delta\phi_1^N + \delta\phi_1^{M\,\prime}\nabla^2\delta\phi_1^{N\,\prime} + \partial\delta\phi_1^{M\,\prime}\partial\delta\phi_1^{N\,\prime})] \ . \end{aligned} \tag{21}$$

On the right hand side of the equation it is the source term. Using the Green's function, $\delta\phi_2^I$ takes the form

$$\delta\phi_2^I(\eta,\mathbf{x}) = \int \frac{\mathrm{d}^3q}{(2\pi)^3}e^{i\mathbf{q}\cdot\mathbf{x}}\left\{\int_{-\infty}^{\eta}\mathrm{d}\tau\int\frac{\mathrm{d}^3k_1\,\mathrm{d}^3k_2}{(2\pi)^6}Gr_q(\eta,\tau)\delta(\mathbf{q}-\mathbf{k_1}-\mathbf{k_2})\mathcal{S}\right\}, \tag{22}$$

where

$$\begin{aligned}\mathcal{S} \;\equiv\;& -a^2 V_{,IJK}\delta\phi_1^J\delta\phi_1^K + \delta_{MN}\mathcal{F}_1\frac{\delta\phi_0^{M\prime}}{\mathcal{H}}\delta\phi_1^N\delta\phi_1^I + \delta_{MN}\mathcal{F}_2\frac{\delta\phi_0^{I\prime}}{\mathcal{H}}\delta\phi_1^M\delta\phi_1^N \\ &+\delta_{MN}\mathcal{G}_1\frac{\delta\phi_0^{M\prime}}{\mathcal{H}}\delta\phi_1^{N\prime}\delta\phi_1^{I\prime} + \delta_{MN}\mathcal{G}_2\frac{\delta\phi_0^{I\prime}}{\mathcal{H}}\delta\phi_1^{M\prime}\delta\phi_1^{N\prime},\end{aligned} \tag{23}$$

and $\{\mathcal{F}_1,\mathcal{F}_2,\mathcal{G}_1,\mathcal{G}_2\}$ are some factors in the momentum space.

$$\mathcal{F}_1 = -2k_2^2\ ,\quad \mathcal{F}_2 = \frac{1}{2}\mathbf{k_1}\cdot\mathbf{k_2} - \frac{1}{(\mathbf{k_1}+\mathbf{k_2})^2}\left(k_1^2k_2^2 + k_1^2\mathbf{k_1}\cdot\mathbf{k_2}\right) \tag{24}$$

$$\mathcal{G}_1 = -\frac{2}{k_1^2}\mathbf{k_1}\cdot\mathbf{k_2}\ ,\quad \mathcal{G}_2 = -\frac{1}{2} + \frac{1}{(\mathbf{k_1}+\mathbf{k_2})^{\mathbf{2}}}\left(k_2^2 + \mathbf{k}_1\cdot\mathbf{k_2}\right) \tag{25}$$

Notice that the terms with $\mathcal{F}_2$ and $\mathcal{G}_2$ are symmetric with M, N, so when we calculate the three-point function, $\mathcal{F}_2$ and $\mathcal{G}_2$ must be symmetrized over permutations of $\{k_1, k_2\}$ as in [28]. On the other hand, $\mathcal{F}_1$ and $\mathcal{G}_1$ cannot be symmetrized.

5 Three-point correlator

The three-point correlator of a free scalar field vanishes, $\langle\delta\phi_1\delta\phi_1\delta\phi_1\rangle = 0$. The leading order of three-point correlator $\langle\delta\phi\delta\phi\delta\phi\rangle$ is $\langle\delta\phi_1\delta\phi_1\delta\phi_2\rangle \sim \frac{1}{2}\langle\delta\phi_1\delta\phi_1\delta\phi_1 * \delta\phi_1\rangle$, where $*$ denotes a convolution. Thus with the value of $\delta\phi_1^I$ (14) and $\delta\phi_2^I$ (22), the three-point correlator of multiple-field can be calculated. As the argument given in [28], the field equation of multiple-field is in the approximation of slow roll limit, and the expansion in powers of slow-roll parameter is not applicable at the end of inflation. The reason is that the the subleading term has logarithmic divergences $\ln|k\eta| = N$, and the growth of the e-folding number makes the subleading terms not negligible. Here we just calculate the three-point correlator when the modes cross the horizon.

According to the source term of the field equation, we will show the results of three-point correlator from the three parts below.

5.1 $V_{,IJK}$ terms

In slow roll approximation, we neglect the $V_{,I}$ and $V_{,IJ}$ terms, but the $V_{,IJK}$ terms could have non-neglectable effect in some situation, and also lead to the logarithmic divergence.

The three-point correlator from $V_{,IJK}$ terms take the form

$$\begin{aligned}\langle\delta\phi^I(\mathbf{k}_1)\delta\phi^J(\mathbf{k}_2)\delta\phi^K(\mathbf{k}_3)\rangle &\supseteq -i(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\int_{-\infty}^{\eta}\mathrm{d}\tau\; a(\tau)^4 V_{,IJK}\times \\ &\Big\{\left[\theta_{k_3}(\tau)\bar{\theta}_{k_3}(\eta)-\bar{\theta}_{k_3}(\tau)\theta_{k_3}(\eta)\right]\bar{\theta}_{k_1}(\eta)\bar{\theta}_{k_2}(\eta)\theta_{k_1}(\tau)\theta_{k_2}(\tau)+ \\ &\left[\theta_{k_2}(\tau)\bar{\theta}_{k_2}(\eta)-\bar{\theta}_{k_2}(\tau)\theta_{k_2}(\eta)\right]\bar{\theta}_{k_1}(\eta)\theta_{k_3}(\eta)\theta_{k_1}(\tau)\bar{\theta}_{k_3}(\tau)+ \\ &\left[\theta_{k_1}(\tau)\bar{\theta}_{k_1}(\eta)-\bar{\theta}_{k_1}(\tau)\theta_{k_1}(\eta)\right]\theta_{k_2}(\eta)\theta_{k_3}(\eta)\bar{\theta}_{k_2}(\tau)\bar{\theta}_{k_3}(\tau)\Big\}. \end{aligned} \tag{26}$$

The θ terms in the brackets come from the Green's function. Outside the brackets, the θ terms which depend on time parameter η are derived from the free scalar field, and the θ terms depending on τ is derived from the source of the field equation. We have stated that θ is independent of the index in multiple-field, so the final result is similar to single field for $V_{,IJK}$ term. With the value of θ (16), this part of three-point correlator can be written to the leading order of slow roll approximation,

$$\begin{aligned}\langle\delta\phi^I(\mathbf{k}_1)\delta\phi^J(\mathbf{k}_2)\delta\phi^K(\mathbf{k}_3)\rangle &\supseteq (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{H_*^2 V_{*,\,IJK}}{4\prod_i k_i^3}\times \\ &\int_{-\infty}^{\eta}\frac{\mathrm{d}\tau}{\tau^4}\,\mathrm{Re}[-i(1-ik_1\tau)(1-ik_2\tau)(1-ik_3\tau)e^{ik_t\tau}]\;, \end{aligned} \tag{27}$$

where $k_t = k_1+k_2+k_3$, $*$ denotes the value at the time η. Here we take η to the value that the modes cross the horizon. Following the paper [5], we can deform the integration variable τ to Euclidean time and deal with the divergence of the integral properly. Finally, we obtain

$$\begin{aligned}\langle\delta\phi^I(\mathbf{k}_1)\delta\phi^J(\mathbf{k}_2)\delta\phi^K(\mathbf{k}_3)\rangle &\supseteq (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{H_*^2 V_{*,IJK}}{4\prod_i k_i^3}\times \\ &\left(-\frac{4}{9}k_t^3+k_t\prod_{i<j}k_ik_j+\frac{1}{3}\left\{\frac{1}{3}+\gamma+\ln|k_t\eta|\right\}\sum_i k_i^3\right)\;, \end{aligned} \tag{28}$$

where $i\in\{1,2,3\}$, and $\gamma\approx 0.577$ is the Euler's constant. In multiple-field, there also exists the infra-red divergence term form the $V_{,IJK}$ terms. When we take the time crossing the horizon, the divergence term is negligible. The classical evolution of perturbation afterwards will make the term large, but we can use other formalism to deal with the problem, such as δN formalism, or the separate universe approach [33, 34].

5.2 $\mathcal{F}$ terms

In this section, we discuss the zero-derivative terms in the source, which contain the contributions of the $\mathcal{F}_1$ and $\mathcal{F}_2$ terms. Since there is a delta function in the three-point correlator, the sum of momentum $\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3$ is zero, we could write $\mathcal{F}_1$ and $\mathcal{F}_2$ in other equivalent form,

$$\mathcal{F}_1(k_1,k_2;k_3) = -2k_2^2\ , \tag{29}$$

and $\mathcal{F}_2$ could be symmetrized as

$$\mathcal{F}_2(k_1,k_2;k_3) = -\frac{1}{2}(k_1^2+k_2^2)+\frac{(k_1^2-k_2^2)^2}{4k_3^2}+\frac{k_3^2}{4}\ . \tag{30}$$

The part of three-point correlator come from the $\mathcal{F}_1$ and $\mathcal{F}_2$ terms is expressed as

$$\begin{aligned}
\langle\delta\phi^I(\mathbf{k}_1)\delta\phi^J(\mathbf{k}_2)\delta\phi^K(\mathbf{k}_3)\rangle \ \supseteq\ & i(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\int_{-\infty}^{\eta}\mathrm{d}\tau\ a(\tau)^2\times\frac{1}{2}\times\\
&\Big\{[\sqrt{2}\epsilon^I\delta^{JK}\mathcal{F}_1(k_1,k_2;k_3)+\sqrt{2}\epsilon^J\delta^{IK}\mathcal{F}_1(k_2,k_1;k_3)+2\sqrt{2}\epsilon^K\delta^{IJ}\mathcal{F}_2(k_1,k_2;k_3)]\\
&\left[\theta_{k_3}(\tau)\bar\theta_{k_3}(\eta)-\bar\theta_{k_3}(\tau)\theta_{k_3}(\eta)\right]\bar\theta_{k_1}(\eta)\bar\theta_{k_2}(\eta)\theta_{k_1}(\tau)\theta_{k_2}(\tau)+\\
&[\sqrt{2}\epsilon^I\delta^{JK}\mathcal{F}_1(k_1,k_3;k_2)+\sqrt{2}\epsilon^K\delta^{IJ}\mathcal{F}_1(k_3,k_1;k_2)+2\sqrt{2}\epsilon^J\delta^{IK}\mathcal{F}_2(k_3,k_1;k_2)]\\
&\left[\theta_{k_2}(\tau)\bar\theta_{k_2}(\eta)-\bar\theta_{k_2}(\tau)\theta_{k_2}(\eta)\right]\bar\theta_{k_1}(\eta)\theta_{k_3}(\eta)\theta_{k_1}(\tau)\bar\theta_{k_3}(\tau)+\\
&[\sqrt{2}\epsilon^J\delta^{IK}\mathcal{F}_1(k_2,k_3;k_1)+\sqrt{2}\epsilon^K\delta^{IJ}\mathcal{F}_1(k_3,k_2;k_1)+2\sqrt{2}\epsilon^I\delta^{JK}\mathcal{F}_2(k_2,k_3;k_1)]\\
&\left[\theta_{k_1}(\tau)\bar\theta_{k_1}(\eta)-\bar\theta_{k_1}(\tau)\theta_{k_1}(\eta)\right]\theta_{k_2}(\eta)\theta_{k_3}(\eta)\bar\theta_{k_2}(\tau)\bar\theta_{k_3}(\tau)\Big\},
\end{aligned} \tag{31}$$

where δ^{IJ} origins from the commutation relation of creation and annihilation operators (15), and the factor 1/2 ahead of the open brace comes from the definition of $\delta\phi_2^I$. Different modes of contracting a free scalar field $\delta\phi_1$ and $\delta\phi_1$ in the source term make $\mathcal{F}_2$ symmetric, and cause the factor 2 in front of $\mathcal{F}_2$. Here the slow-roll parameter ϵ^I is defined in (6).

In the three-point correlator, $\mathcal{F}$ terms are independent of integration variable, so the factors of final results f_1, f_2, and f_3 are similar to the case of single field,

$$\begin{aligned}
\langle\delta\phi^I(\mathbf{k}_1)\delta\phi^J(\mathbf{k}_2)\delta\phi^K(\mathbf{k}_3)\rangle \ \supseteq\ & (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{H_*^4}{8\prod_i k_i^3}\times\frac{1}{2}\\
&\Big\{f_1[\sqrt{2}\epsilon^I\delta^{JK}\mathcal{F}_1(k_1,k_2;k_3)+\sqrt{2}\epsilon^J\delta^{IK}\mathcal{F}_1(k_2,k_1;k_3)+2\sqrt{2}\epsilon^K\delta^{IJ}\mathcal{F}_2(k_1,k_2;k_3)]+\\
&f_2[\sqrt{2}\epsilon^I\delta^{JK}\mathcal{F}_1(k_1,k_3;k_2)+\sqrt{2}\epsilon^K\delta^{IJ}\mathcal{F}_1(k_3,k_1;k_2)+2\sqrt{2}\epsilon^J\delta^{IK}\mathcal{F}_2(k_3,k_1;k_2)]+\\
&f_3[\sqrt{2}\epsilon^J\delta^{IK}\mathcal{F}_1(k_2,k_3;k_1)+\sqrt{2}\epsilon^K\delta^{IJ}\mathcal{F}_1(k_3,k_2;k_1)+2\sqrt{2}\epsilon^I\delta^{JK}\mathcal{F}_2(k_2,k_3;k_1)]\Big\},
\end{aligned} \tag{32}$$

where

$$
\begin{aligned}
f_1 &\equiv -\frac{2k_3^3(k_1^2+4k_1k_2+k_2^2-k_3^2)}{(k_1+k_2-k_3)^2k_t^2}\,, \\
f_2 &\equiv -\frac{2k_2^3(k_1^2-4k_1k_3+k_3^2-k_2^2)}{(k_2^2-(k_1-k_3)^2)^2}\,, \\
f_3 &\equiv -\frac{2k_1^3(k_2^2+4k_2k_3+k_3^2-k_1^2)}{(k_1-k_2-k_3)^2k_t^2}\,.
\end{aligned} \tag{33}
$$

The factor H_*^4 origins from θ and the scale factor a.

5.3 $\mathcal{G}$ terms

Similar to the derivation of $\mathcal{F}$ terms, $\mathcal{G}$ terms lead to another part of the three-point correlator. We have

$$
\mathcal{G}_1(k_1,k_2,k_3) = \frac{k_1^2+k_2^2-k_3^2}{k_1^2}\ , \tag{34}
$$

and $\mathcal{G}_2 = 0$ when symmetrized. The expectation value of $\mathcal{G}$ terms is

$$
\begin{aligned}
\langle\delta\phi^I(\mathbf{k}_1)\delta\phi^J(\mathbf{k}_2)\delta\phi^K(\mathbf{k}_3)\rangle \ \supseteq\ & i(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\int_{-\infty}^{\eta}\mathrm{d}\tau\ a(\tau)^2\times\frac{1}{2}\times \\
& \{[\sqrt{2}\epsilon^I\delta^{JK}\mathcal{G}_1(k_1,k_2;k_3)+\sqrt{2}\epsilon^J\delta^{IK}\mathcal{G}_1(k_2,k_1;k_3)] \\
& \left[\theta_{k_3}(\tau)\bar\theta_{k_3}(\eta)-\bar\theta_{k_3}(\tau)\theta_{k_3}(\eta)\right]\bar\theta_{k_1}(\eta)\bar\theta_{k_2}(\eta)\frac{\mathrm{d}}{\mathrm{d}\tau}\theta_{k_1}(\tau)\frac{\mathrm{d}}{\mathrm{d}\tau}\theta_{k_2}(\tau)+ \\
& [\sqrt{2}\epsilon^I\delta^{JK}\mathcal{G}_1(k_1,k_3;k_2)+\sqrt{2}\epsilon^K\delta^{IJ}\mathcal{G}_1(k_3,k_1;k_2)] \\
& \left[\theta_{k_2}(\tau)\bar\theta_{k_2}(\eta)-\bar\theta_{k_2}(\tau)\theta_{k_2}(\eta)\right]\bar\theta_{k_1}(\eta)\theta_{k_3}(\eta)\frac{\mathrm{d}}{\mathrm{d}\tau}\theta_{k_1}(\tau)\frac{\mathrm{d}}{\mathrm{d}\tau}\bar\theta_{k_3}(\tau)+ \\
& [\sqrt{2}\epsilon^J\delta^{IK}\mathcal{G}_1(k_2,k_3;k_1)+\sqrt{2}\epsilon^K\delta^{IJ}\mathcal{G}_1(k_3,k_2;k_1)] \\
& \left[\theta_{k_1}(\tau)\bar\theta_{k_1}(\eta)-\bar\theta_{k_1}(\tau)\theta_{k_1}(\eta)\right]\theta_{k_2}(\eta)\theta_{k_3}(\eta)\frac{\mathrm{d}}{\mathrm{d}\tau}\bar\theta_{k_2}(\tau)\frac{\mathrm{d}}{\mathrm{d}\tau}\bar\theta_{k_3}(\tau)\}.
\end{aligned} \tag{35}
$$

Repeating the progress in the case of $\mathcal{F}$ terms, we obtain

$$
\begin{aligned}
\langle\delta\phi^I(\mathbf{k}_1)\delta\phi^J(\mathbf{k}_2)\delta\phi^K(\mathbf{k}_3)\rangle \ \supseteq\ & (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{H_*^4}{8\prod_i k_i^3}\times\frac{1}{2}\times \\
& \Big\{g_1[\sqrt{2}\epsilon^I\delta^{JK}\mathcal{G}_1(k_1,k_2;k_3)+\sqrt{2}\epsilon^J\delta^{IK}\mathcal{G}_1(k_2,k_1;k_3)]+ \\
& g_2[\sqrt{2}\epsilon^I\delta^{JK}\mathcal{G}_1(k_1,k_3;k_2)+\sqrt{2}\epsilon^K\delta^{IJ}\mathcal{G}_1(k_3,k_1;k_2)]+ \\
& g_3[\sqrt{2}\epsilon^J\delta^{IK}\mathcal{G}_1(k_2,k_3;k_1)+\sqrt{2}\epsilon^K\delta^{IJ}\mathcal{G}_1(k_3,k_2;k_1)]\Big\}\ ,
\end{aligned} \tag{36}
$$

where

$$
\begin{aligned}
g_1 &\equiv \frac{4k_3 \prod_i k_i^2}{(k_1+k_2-k_3)^2 k_t^2}\,, \\
g_2 &\equiv \frac{4k_2 \prod_i k_i^2}{(k_2^2-(k_1-k_3)^2)^2}\,, \\
g_3 &\equiv \frac{4k_1 \prod_i k_i^2}{(k_1^2-(k_2+k_3)^2)^2}\,.
\end{aligned} \tag{37}
$$

Then we find the summation of the three-point correlator. For simplicity, the $V_{,IJK}$ terms are neglected.

$$
\begin{aligned}
\langle \delta\phi^I(\mathbf{k}_1)\delta\phi^J(\mathbf{k}_2)\delta\phi^K(\mathbf{k}_3)\rangle \supseteq\ & (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\times \\
& \frac{2\pi^4 \mathrm{P}_*^2}{\prod_i k_i^3}\sum_{perms}\frac{\dot{\phi}_*^I}{H_*}\delta^{JK}\mathcal{A}(k_1,k_2,k_3),
\end{aligned} \tag{38}
$$

$$
\mathcal{A}(k_1,k_2,k_3) = \frac{1}{2}k_1^3 - \frac{k_1(k_2^2+k_3^2)}{2} - \frac{4}{k_t}k_2^2k_3^2\,, \tag{39}
$$

where $\mathrm{P}_* = H_*^2/4\pi^2$ is the power spectrum of scalar field when modes of scalar field perturbation cross the horizon, and $\mathcal{A}(k_1,k_2,k_3)$ is the shape factor of the three-point correlator. The result is easily reduced to the case of single field, and consistent with [5, 28]. [1]

6 Non-Gaussianity $f_{\rm NL}$

The non-Gaussianity of multiple-field inflation is shown in this section. The inflaton will decay at the end of inflation, and the final observable is the curvature perturbation ζ. To discuss the non-Gaussianity, the power spectrum and bispectrum of curvature are given,

$$
\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle \equiv (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2)\frac{2\pi^2}{k_1^3}\mathrm{P}_\zeta(k_1)\,, \tag{40}
$$

$$
\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_1)\rangle \equiv (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_\zeta(k_1,k_2,k_3)\,. \tag{41}
$$

The non-Gaussianity is the deviation from the Gaussian statistics in CMB, and use the parameter $f_{\rm NL}$ to represent its magnitude,

$$
\zeta = \zeta_g + \frac{3}{5}f_{\rm NL}\left(\zeta_g^2 - \langle\zeta_g^2\rangle\right)\,, \tag{42}
$$

[1]Since we have used the metric of field space δ^{JK}, the result is a little different from [27]. Therefore, k_2 and k_3 are symmetric here. If we symmetrize k_2 and k_3 in eq.(69) of [27], then we get the same result. So both results are equivalent.

where ζ_g denotes the Gaussian part of ζ. [2] Using this definition, we obtain

$$\frac{6}{5}f_{\rm NL} = \frac{\prod_i k_i^3}{\sum_i k_i^3}\frac{B_\zeta}{4\pi^4 {\rm P}_\zeta^2}\ . \tag{43}$$

Notice that the field equation formalism which is used to calculate the non-Gaussianity is applicable when the modes cross the horizon. In multiple-field inflation, there exists entropy perturbation. Thus in order to consider the effects afterward, the δN formalism is a good method. On large scale, the value of curvature perturbation is the e-folding number from the initial flat slice at t_* to the final uniform density slice at time t,

$$\zeta(t,\mathbf{x}) \simeq \delta N = N(t,t_*,\mathbf{x}) - N(t,t_*)\ , \tag{44}$$

where the e-folding number is defined as

$$N(t,t_*) \equiv \int_{t_*}^{t} H{\rm d}t\ . \tag{45}$$

δN can be expanded by the initial scalar fields,

$$\delta N = N_{,I}\delta\phi^I + \frac{1}{2}N_{,IJ}\delta\phi^I\delta\phi^J + \cdots\ . \tag{46}$$

The power spectrum and bispectrum can be expressed by δN formalism,

$$\begin{aligned} {\rm P}_\zeta &= \delta^{IJ}N_{,I}N_{,J}{\rm P}_*\ , && (47)\\ \langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle &= N_{,I}N_{,J}N_{,K}\langle\delta\phi^I(\mathbf{k}_1)\delta\phi^J(\mathbf{k}_2)\delta\phi^K(\mathbf{k}_3)\rangle + \\ &\quad \frac{1}{2}N_{,I}N_{,J}N_{,KL}\langle\delta\phi^I(\mathbf{k}_1)\delta\phi^J(\mathbf{k}_2)(\delta\phi^K * \delta\phi^L)(\mathbf{k}_3)\rangle + perms\ , && (48)\end{aligned}$$

where * denotes a convolution and the higher order terms are neglected. The non-linear parameter $f_{\rm NL}$ is derived from (43), (47) and (48),

$$f_{\rm NL} = \frac{5{\rm P}_*}{12{\rm P}_\zeta}\frac{1}{\sum_i k_i^3}\Big(-\frac{1}{2}\sum_i k_i^3 + \frac{4}{k_t}\sum_{i<j}k_i^2k_j^2 + \frac{1}{2}\sum_{i\neq j}k_ik_j^2\Big) + \frac{5}{6}\frac{N_{,I}N_{,J}N_{,IJ}}{(\delta^{IJ}N_{,I}N_{,J})^2}\ . \tag{49}$$

The last term on the right hand side is from the curvature evolution on large scale, which contributes the local form of non-Gaussianity. The equilateral shape of non-Gaussianity from multiple-field is constrained by the tensor-to-scalar ratio $r \sim {\rm P}_*/{\rm P}_\zeta$.

[2] Here the sign of $f_{\rm NL}$ is consistent with the convention of the CMB experiments, and different from the paper written by Maldacena [5].

7 Conclusion

In this paper, we derive the second-order field equation of multiple-field, and calculate the shape of non-Gaussianity for multiple-field inflation with the method of the field equation. The shape of Non-Gaussiantiy derives from the three-point correlator, which implies the microphysics in the period of inflation. Our result of the three-point correlator is consistent with the previous one [27] which uses the method of in-in formalism. And it is easy to extend the Bunch-Davies vacuum to the α vacuum in the field equation formalism which shows the trans-Plankian physics [35] from non-Gaussianity.

The field equation formalism is applicable when we know the equation of motion, even if the action is not given. Meanwhile, we should notice that the formalism is used when the modes of scalar field perturbation crossing the horizon. After crossing the horizon, the quantum fluctuations become classical due to decoherence [36]. Then the classical evolution of curvature perturbation on large scale could be solved by the δN formalism. Finally, we could get the non-Gaussianity observed in the CMB.

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